The smallest 4-regular 4-chromatic graphs with girth 5

Gunnar Brinkmann
Fakultät für Mathematik
Universität Bielefeld
D-33501 Bielefeld
Germany

Markus Meringer
Lehrstuhl II für Mathematik
Universität Bayreuth
D-95440 Bayreuth
Germany

November 12, 1996

Abstract

In this note we give the smallest 4-regular 4-chromatic graphs with girth 5. There are exactly one graph on 21 vertices and one on 25 vertices.

Starting with a conjecture of Grünbaum [2] saying that (k, k, g)-graphs (that is k-chromatic k-regular graphs with girth at least g) exist for all $k \geq 2$ and $g \geq 4$ a lot of work has been done investigating such graphs.
In [4] Song Wenjie et.al. show that $(4,4,4)$ graphs of order n exist if and only if $n >= 12$. Furthermore they define $f(4,4,g)$ to be the smallest number of vertices so that a $(4,4,g)$ graph exists and conjecture that $f(4,4,5) = 25$. This conjecture is false. In [1] the first author lists the $(4,4,5)$ graph on 21 vertives given in figure one. The graph was constructed with the help of a computer, and it was also shown that it is the smallest possible $(4,4,5)$ graph. So we have $f(4,4,5) = 21$. A further result was that there are no $(4,4,5)$ graphs on 22 or 23 vertices. Using his completely independent generation program *genreg*, the second author [3] verified these results and extended them showing that there are no $(4,4,5)$ graphs on 24 vertices and exactly one $(4,4,5)$ graph on 25 vertices (given in figure 2). He also proved that $f(4,4,6) > 34$. In fact with only 2 exceptions all 4-regular graphs with girth 6 on up to 34 vertices turned out to be bipartite.
Both $(4,4,5)$ graphs become 3-colourable when an edge is deleted, so it is impossible to construct an infinite series of $(4,4,5)$ graphs using the method from [4].

References

[1] G. Brinkmann. Generating cubic graphs faster than isomorphism checking. Preprint SFB 343 No. 92-047, 1992. Bielefeld.

[2] B. Grünbaum. A problem in graph colouring. *Amer. Math. Monthly*, 77:1088–1092, 1970.

[3] M. Meringer. Erzeugung regulärer Graphen. Master's thesis, Universität Bayreuth, 1996.

[4] S. Wenjie, W. Jianzhong, Y. Tianxing, and Z. Kemin. A note on 4-regular 4-chromatic graphs with girth 4. *Graph Theory Notes of New York*, XXX(9):35–36, 1996.

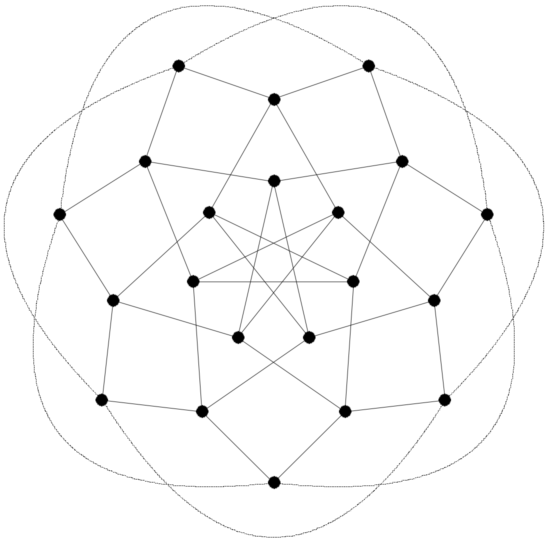

Figure 1: The $(4,4,5)$ graph on 21 vertices

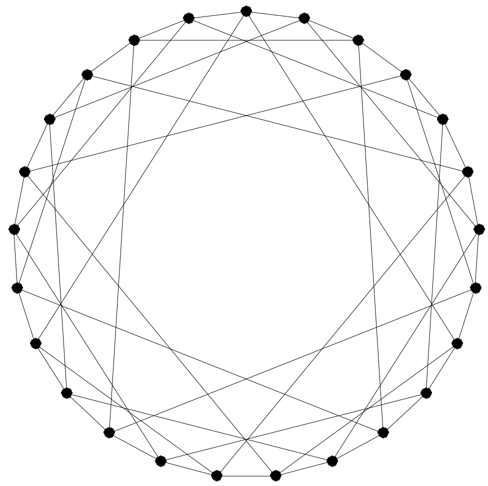

Figure 2: The $(4,4,5)$ graph on 25 vertices